Large eddy simulation of turbulent flow in porous media

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Abstract

An LES (large eddy simulation) study was conducted using one of standard numerical models for a porous medium, namely, a flow through a periodic array of square cylinders. The LES results were processed to extract macroscopic results such as the macroscopic turbulent kinetic energy and the macroscopic pressure gradient. These macroscopic results are compared against those obtained using conventional models of turbulent kinetic energy and its dissipation rate, so as to examine the validity of extending the conventional two equation models of turbulence to the flow in porous media. The spectrum of turbulence was also examined to appreciate the onset of turbulence.

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1. Introduction

Kuwahara et al. [1], Nakayama and Kuwahara [2], De Lemos and Pedras [3] conducted numerical experiments for turbulent flows through a periodic array of cylinders using the conventional two-equation turbulence model based on Reynolds Averaged Navier–Stokes (RANS) equations. The pressure results from their numerical experiments showed a good agreement with Forchheimer-extended Darcy's law. This prompted Nakayama and Kuwahara [2] to conclude that Forchheimer extended-Darcy's law holds even in the turbulent flow regime in porous media. However, the application of conventional two-equation turbulence models to such complex unsteady turbulent flows as in porous media must be re-examined for a further discussion related to turbulence in porous media.

In contrast to statistical turbulence models, direct numerical simulations (DNS) require no assumptions, but will not be available for engineering applications for some time, because of large memory and CPU time requirements. Large eddy simulation (LES) method can be described as compromise between the DNS and the simulation based on RANS equations. For the LES, the three-dimensional unsteady Navier–Stokes equations are filtered in order to separate the large scale eddies from the small scale ones. Thus, the main flow structure is resolved directly, while only the small scale eddies are modeled by a subgrid scale model.

The LES study is presented below for unsteady flows through periodic arrays of square cylinders. The macroscopic results based on the LES solutions are compared against the previous results based on RANS with the two-equation...
turbulence model, so as to examine the validity of the simulation procedure using RANS with a two-equation turbulence model, for obtaining the macroscopic quantities associated with turbulent flow in porous media.

2. Numerical model and scheme

The physical model for a porous medium is illustrated in Fig. 1, where a periodic array of square cylinders is shown. In this LES study, only one structural unit is chosen for a calculation domain, and along its boundaries, the periodicity of the flow is assumed. In this way, we intentionally neglect the eddies larger than the scale of porous structure, since such large eddies cannot survive long enough to be detected in the porous medium. (It should, however, be noted that vortices larger than the scale of porous structure may appear when the cylinders are closely placed.) Furthermore, the Renormalization Group (RNG) subgrid scale model [4] is used for modeling non-resolvable subgrid scale eddies. In highly turbulent regions, the RNG subgrid scale model reduces to the Smagorinsky model [5], while, in weakly turbulent regions, it correctly yields zero SGS viscosity without any ad hoc modifications.

As for the advection terms, QUICK scheme is introduced. The code used to solve the filtered equations is based on a three-dimensional finite volume method [6]. The Reynolds number $Re_H = u_D H / \nu$ based on the center-to-center distance $H$ was varied from 100 to $5 \times 10^4$, while the porosity $\phi = 1 - (D / H)^2$ was changed from 0.3 to 0.9. Computation were carried out using grid systems, which guarantee that the grid spacing close to the wall is small enough to satisfy the condition $\Delta y < (\nu^3 / \varepsilon)^{1/4}$. A typical grid system for computation consists of $161 \times 91 \times 35 = 512,785$ nodes with highly non-uniform grid spacing to cover the domain of integration $2H \times H \times 0.5H$. Preliminary calculations were made to compare the results against those obtained using $243 \times 137 \times 35 = 1,165,185$ nodes for some selected cases. In this way, the originally used grid resolution was found sufficient. Moreover, in order to investigate the effect of its spanwise size on the LES results, the domain of integration was doubled in the spanwise direction by doubling the number of grid points, while keeping the spanwise cell size constant. This modification yielded no significant changes in the LES results, proving that the originally used domain is fully sufficient. The time step was set small enough to satisfy Courant condition, $\Delta t < \min(\Delta x, \Delta y, \Delta z) / \langle u \rangle^f$ (where $\langle u \rangle^f$ is the time averaged intrinsic velocity) after confirming that any further decrease in the time step does not alter the results significantly. A typical time period advanced in the LES was $20H / \langle u \rangle^f$, which was found long enough for the flow to develop periodically.

3. Velocity fluctuations and turbulent kinetic energy

Fig. 2 shows a series of the spanwise velocity oscillations at a selected point behind the cylinder for the case of $\phi = 0.64$. The spanwise oscillations indicative of three-dimensional velocity fluctuations (i.e. turbulent motions)
are already appreciable at $Re_D = u_D D / v = 80$. The resulting three normal stresses are found to be of the same order of magnitude. The turbulent kinetic energy for $Re_D = 400$ is illustrated in the spectrum in Fig. 3, where the $-5/3$ law for the inertial subrange may be confirmed. The figure indicates unreasonably high generation of the turbulent kinetic energy at high frequencies. This kind of white noise inevitably appears in the spectra of the present LES.

Fig. 2. Spanwise velocity oscillations for $\phi = 0.64$.

Fig. 3. Spectrum of turbulent kinetic energy at $Re_D = 400$. 
In Fig. 4, the time-averaged velocity, pressure and turbulent kinetic energy fields for \(Re_D=10^4\) based on the LES (presented on the left) are compared against those based on RANS with a low Reynolds number version of the \(k-e\) model (presented on the right). The figure shows that the mean velocity and pressure fields from both LES and RANS are in reasonable agreement, while the turbulent kinetic energy field of the LES is quite different from that of RANS with the \(k-e\) model. It is well known that conventional \(k-e\) models coupled with the effective viscosity formulation tend to overestimate the level of turbulent kinetic energy around the stagnation point. The reason for this may best be illustrated by writing the production rate of \(k\) for the \(k-e\) model:

\[
P = \nu \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

where \(\nu\) and \(u_i\) are the eddy diffusivity and time averaged velocity vector, respectively.

The foregoing \(k\) production term contains such a predominant term as \((\partial u / \partial x)^2\) that attains a quite large value for a decelerating flow around the front stagnation point, yielding extremely high production rate of \(k\) there. This shortcoming inherent to the conventional \(k-e\) models has been remedied by the LES, which provides a reasonable turbulent kinetic energy distribution. It can clearly be seen from the figure that the turbulent kinetic energy is produced exclusively within the shear layers above the lateral surfaces of the cylinder, where the mean strain rate is quite high, due to flow separation.

A series of computations carried out for various sets of porosity and Reynolds number are presented in Fig. 5(a) in terms of the macroscopic turbulent kinetic energy \(\langle k \rangle^f\) (i.e. the volume averaged turbulent kinetic energy) normalized by the square of Darcian velocity. As found in the previous study [2], the normalized \(\langle k \rangle^f\) increases with \(Re_D\), and stays constant for large \(Re_D\), say, \(Re_D > 3000\). The figure again suggests that turbulence may appear at comparatively low Reynolds number especially in the case of small porosity, as pointed out by Masuoka [7]. Since \(u_D^2 = (\phi \langle u \rangle^f)^2\) can be quite small, the normalized \(\langle k \rangle^f\) exceeds unity for comparatively small \(\phi\). (Note that \(\langle k \rangle^f / (\langle u \rangle^f)^2\) never exceeds 0.5.)
The constant values of $\langle k \rangle_f$ for large $Re_D$ are plotted in Fig. 5(b) together with those obtained using RANS with the low Reynolds number version of the $k$–$\epsilon$ model. The present results from the LES are somewhat lower than those based on the correlation proposed by Nakayama and Kuwahara [2], namely,

$$\langle k \rangle_f = 3.7 \frac{1-\phi}{\sqrt{\psi}} u_D^2,$$

(2)

Despite of the shortcoming associated with the production term, the low Reynolds number version of the $k$–$\epsilon$ model gives a reasonable level of macroscopic turbulent kinetic energy. Thus, the conventional two-equation turbulence model based on Reynolds Averaged Navier–Stokes (RANS) equations may well be used to estimate the macroscopic field of turbulent flow in porous media. Furthermore it should be noted that, unlike in clear fluid flows, the turbulent Reynolds number $\langle k \rangle_f/\langle \epsilon \rangle_f$ is of the same order of magnitude as the macroscopic Reynolds number $Re_D$.

4. Macroscopic pressure gradient in turbulent flow

The results of the macroscopic pressure gradient calculated using the LES results are presented in a dimensionless fashion against $Re_D$ for the case of $\phi=0.84$ in Fig. 6, where the previous results based the low Reynolds number version of the $k$–$\epsilon$ model are also plotted for comparison. Agreement between the results based on LES and those based on the $k$–$\epsilon$ model appears excellent, which substantiates the validity of the simulation using RANS with a two-equation turbulence model.

Fig. 5. Macroscopic turbulent kinetic energy.

Fig. 6. Effect of Reynolds number on macroscopic pressure gradient.
The functional relationship of the Forchheimer drag is investigated by plotting the dimensionless pressure gradient $-\left(\frac{d(p')}{dx}\right)\left(\frac{D/\rho u_D^2}{\rho}\right)$, using only the high Reynolds number results obtained at $Re_D>3000$. Ergun’s empirical equation accounting for the Forchheimer drag in packed beds of particle diameter $d_p$ is given by

$$-\frac{d(p')}{dx}\left(\frac{d_p}{\rho u_D^2}\right) = \frac{150(1-\phi)^2}{\phi} \left(\frac{v}{u_D d_p}\right) + 1.75 \frac{1-\phi}{\phi^3} \equiv 1.75 \frac{1-\phi}{\phi^3} (Re_{d_p}>3000).$$

This prompts us to correlate the foregoing dimensionless pressure gradient with $(1-\phi)/\phi^3$, anticipating a linear relationship with $(1-\phi)/\phi^3$. Thus, the pressure gradient results are re-processed and plotted in Fig. 7, which, in fact, substantiate a linear relationship as

$$-\frac{d(p')}{dx}\left(\frac{D}{\rho u_D^2}\right) = 2.0 \frac{1-\phi}{\phi^3} (Re_D>3000).$$

This value of 2.0 for a periodic array of square cylinders of size $D$ is slightly higher than 1.75 in Ergun’s Eq. (3) for a packed bed with a particle diameter $d_p=D$.

The foregoing high Reynolds number results may be coupled with the low Reynolds number results previously established by Kuwahara et al. [1] for laminar Darcy’s flow, conducting the numerical experiment for the same geometrical array, namely,

$$-\frac{d(p')}{dx} = \frac{120(1-\phi)^2}{\phi^3 D^2} \mu u_D \text{ (Darcy flow)}$$

so that the corresponding law for the relation between the friction coefficient and Reynolds number is given by

$$\lambda_{eq} = \left(-\frac{d(p')}{dx}\right) \left(\frac{\rho((u')^2)}{2d_{eq}}\right) = \frac{51.2}{Re_{d_eq}} + 1.85$$

where

$$Re_{d_eq} = \frac{(u')^2 d_{eq}}{v} = \frac{(u')^2}{v} \left(\frac{32}{150} \frac{\phi}{1-\phi} \frac{D}{v}\right)$$

is the equivalent Reynolds number which transforms Ergun’s Eq. (3) (for $d_p=D$) into Poiseuille’s expression:

$$\lambda_{eq} = \left(-\frac{d(p')}{dx}\right) \left(\frac{\rho((u')^2)}{2d_{eq}}\right) = \frac{64}{Re_{d_eq}} + 1.62$$

Fig. 7. Effect of porosity on Forchheimer drag.

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Fig. 7. Effect of porosity on Forchheimer drag.
Finally, the present Eq. (6) obtained for the array of square cylinders is compared with Ergun's Eq. (8) in Fig. 8. Both equations are quite close to each other. This indicates that Ergun's Eq. (8) for the relationship between the pressure and Reynolds number is such a universal equation that it can be used for most of two- and three-dimensional periodic structures in a wide porosity range.

5. Conclusions

The present LES study revealed that Ergun's equation may well describe the drag relationship for the turbulent flow in porous media. Furthermore, it suggests that turbulence may appear in porous media at comparatively low Reynolds number. However, it must be pointed out that the characteristics of turbulence in porous media may differ significantly from those in clear fluid flow, because of comparatively high turbulence intensity usually encountered in such porous media. Further numerical experiments (preferably DNS) are definitely needed to explore how unsteady laminar flow develops into turbulent flow and how the onset of turbulence influences the drag characteristics.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>D</td>
<td>Size of square cylinder</td>
</tr>
<tr>
<td>$d_{eq}$</td>
<td>Equivalent tube diameter</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Particle diameter</td>
</tr>
<tr>
<td>$k$</td>
<td>Turbulent kinetic energy</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$Re_D = \frac{u_D D}{\nu}$</td>
<td>Reynolds number based on Darcian velocity and $D$</td>
</tr>
<tr>
<td>$Re_{deq} = \frac{(u') d_{eq}}{\nu}$</td>
<td>Reynolds number based on intrinsic velocity and equivalent tube diameter</td>
</tr>
<tr>
<td>$Re_H = \frac{u_D H}{\nu}$</td>
<td>Reynolds number based on Darcian velocity and $H$</td>
</tr>
<tr>
<td>$u$, $v$, $w$</td>
<td>Microscopic velocity components in the $x$, $y$ and $z$ directions</td>
</tr>
<tr>
<td>$u_D$</td>
<td>Darcian velocity</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Cartesian coordinate</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Turbulent kinetic energy dissipation rate</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Porosity</td>
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</tbody>
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Fig. 8. Universal laws for macroscopic pressure gradient.
Special symbols

\[ \langle \rangle \quad \text{volume average} \\
\langle f \rangle \quad \text{intrinsic average} \\

\textbf{References}