

# A Local Thermal Non-Equilibrium Analysis of Fully Developed Forced Convective Flow in a Tube Filled with a Porous Medium

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**Abstract** A local thermal non-equilibrium model has been considered for the case of thermally fully developed flow within a constant heat flux tube filled with a porous medium. Exact temperature profiles for the fluid and solid phases are found after combining the two individual energy equations and then transforming them into a single ordinary differential equation with respect to the temperature difference between the solid phase and the wall subject to constant heat flux. The exact solutions for the case of metal-foam and air combination reveal that the local thermal equilibrium assumption may fail for the case of constant heat flux wall. The Nusselt number is presented as a function of the Peclet number, which shows a significant increase due to both high stagnant thermal conductivity and thermal dispersion resulting from the presence of the metal-foam.

**Keywords** Porous media · Thermal non-equilibrium · Aluminum · Metal foam · Conductivity

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### List of Symbols

$A$	Surface area ( $\text{m}^2$ )
$A_{\text{int}}$	Interface between the fluid and solid ( $\text{m}^2$ )
$c$	Specific heat ( $\text{J/kg K}$ )
$c_p$	Specific heat at constant pressure ( $\text{J/kg K}$ )
$D$	Tube diameter
$d_m$	Mean pore diameter (m)
$h_v$	Volumetric heat transfer coefficient ( $\text{W/m}^3 \text{K}$ )
$k$	Thermal conductivity ( $\text{W/m K}$ )
$K$	Permeability ( $\text{m}^2$ )
$n_j$	Unit vector pointing outward from the fluid side to solid side (–)
$Pr$	Prandtl number (–)
$q$	Heat flux ( $\text{W/m}^2$ )
$r$	Radial coordinate
$T$	Temperature (K)
$u_D$	Darcian velocity (uniform inlet velocity) (m/s)
$u_i$	Velocity vector (m/s)
$V$	Representative elementary volume ( $\text{m}^3$ )
$x_i$	Cartesian coordinates (m)
$x, y, z$	Cartesian coordinates (m)

### Greek Symbols

$\varepsilon$	Porosity (–)
$\varepsilon^*$	Effective porosity (–)
$\nu$	Kinematic viscosity ( $\text{m}^2/\text{s}$ )
$\rho$	Density ( $\text{kg/m}^3$ )
$\tilde{\phi}$	Deviation from intrinsic average
$\langle \phi \rangle$	Darcian average
$\langle \phi \rangle^{f,S}$	Intrinsic average

### Subscripts and Superscripts

dis	Dispersion
f	Fluid
s	Solid
stag	Stagnation
w	Wall

## 1 Introduction

A number of researchers such as Quintard (1998), Quintard and Whitaker (1993), Quintard and Whitaker (1995), Haddad et al. (2004), Hsu (2000), Hsu et al. (1995), and Alazmi and Vafai (2002) investigated heat transfer within fluid saturated porous media and noted that there are many physical situations in which the local thermal equilibrium assumption fails. They pointed out shortcomings of local thermal equilibrium models (i.e., one energy equation models) and proposed local thermal non-equilibrium models (i.e., two energy equation

models). In the thermal equilibrium models, local thermal equilibrium between the solid and fluid phases is assumed whereas such assumption is discarded in the local thermal non-equilibrium models. Extensive discussions on the validity and assessment of the local thermal equilibrium assumptions were made by a substantial number of researchers such as [Minkowycz et al. \(1999\)](#), [Kim and Jang \(2002\)](#), [Al-Nimr and Kiwan \(2002b\)](#), [Al-Nimr and Abu-Hijleh \(2002a\)](#), [Abu-Hijleh et al. \(2002\)](#), [Khashan et al. \(2005\)](#), and [Khashan and Al-Nimr \(2005\)](#). Concise yet comprehensive discussion on the topic may be found in [Nield and Bejan \(2006\)](#).

Some analytical investigations on the local thermal non-equilibrium problems are available in which the exact solutions based on the local thermal equilibrium have been examined by comparing the results against those based on a two equation model. [Kuznetsov \(1996, 1997\)](#) presented elegant perturbation solutions for sensible heat storage packed beds. [Nakayama et al. \(2001\)](#) used the two energy equation model introduced by [Hsu \(2000\)](#) and [Hsu et al. \(1995\)](#), and obtained exact solutions for two fundamental steady heat transfer cases, namely, one-dimensional steady heat conduction in a porous slab with internal heat generation, and also thermally developing unidirectional flow through a semi-infinite porous medium. They pointed out that the thermal equilibrium assumption ceases to be valid even for certain steady thermal problems. Recently, [Kuznetsov and Nield \(2010\)](#) investigated the onset of convection in a horizontal layer of a porous medium saturated by a nanofluid and reported that the effect of the local thermal equilibrium can be quite significant in some circumstances.

Many investigators, who worked on two equation models, neglected the effects of tortuosity on the stagnant thermal conductivity, and simply evaluated the fluid phase thermal conductivity as a product of the porosity and its thermal conductivity, and likewise for the solid phase thermal conductivity. Such evaluations lead to significant errors in thermal conduction especially when the solid thermal conductivity is much higher than the fluid thermal conductivity, such as in the case of metal foams. [Yang and Nakayama \(2010\)](#) proposed a general two-energy equation model, which takes account of both tortuosity and thermal dispersion. [Kuwahara et al. \(2011\)](#) recently proposed an effective porosity concept, which reduces their general two-equation model in a concise form and applied it to two fundamental plane convection cases, namely, convection in a channel filled with a metal foam bounded by isothermal walls and also that bounded by constant heat flux walls. However, none of axi-symmetric cases were treated.

In this study, we shall treat what seems to be one of the most fundamental yet important axi-symmetric cases in practical applications, namely, the thermally fully developed flow in a circular tube filled with a porous medium ([Yang et al. 2009](#)). An aluminum foam and air combination is treated in consideration of possible practical applications in heat exchanging systems. We shall examine the general two-energy equation model introduced by [Yang and Nakayama \(2010\)](#) and seek possible exact solutions for this particular fundamental problem, exploiting the effective porosity concept. These exact solutions may be quite useful for the benchmark tests of numerical tools based on the thermal non-equilibrium assumption. An elegant integral treatment will also be presented to obtain an approximate yet useful expression for the Nusselt number. It will be shown that the case of constant wall heat flux must be treated by using a thermal non-equilibrium model, since the fluid and solid phases within the channel are never at thermal equilibrium.

### 2 Local Thermal Non-Equilibrium Model

Yang and Nakayama (2010) considered the two energy equations for the fluid and solid matrix phases, respectively, and integrated them over a local control volume using the volume averaging theory (Quintard and Whitaker 1993, 1995; Cheng 1978; Nakayama 1995). The resulting volume averaged energy equations run as follows:

For the fluid phase:

$$\begin{aligned} \varepsilon \rho_f c_{pf} \frac{\partial \langle T \rangle^f}{\partial t} + \varepsilon \rho_f c_{pf} \frac{\partial \langle u_j \rangle^f \langle T \rangle^f}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \varepsilon k_f \frac{\partial \langle T \rangle^f}{\partial x_j} + \frac{k_f}{V} \int_{A_{int}} T n_j dA - \rho_f c_{pf} \varepsilon \langle \tilde{u}_j \tilde{T} \rangle^f \right) \\ + \frac{1}{V} \int_{A_{int}} k_f \frac{\partial T}{\partial x_j} n_j dA \end{aligned} \tag{1}$$

For the solid matrix phase:

$$\rho_s c_s (1 - \varepsilon) \frac{\partial \langle T \rangle^s}{\partial t} = \frac{\partial}{\partial x_j} \left( (1 - \varepsilon) k_s \frac{\partial \langle T \rangle^s}{\partial x_j} - \frac{k_s}{V} \int_{A_{int}} T n_j dA \right) - \frac{1}{V} \int_{A_{int}} k_f \frac{\partial T}{\partial x_j} n_j dA \tag{2}$$

where the volume average of a certain variable  $\phi$  in the fluid phase is defined as

$$\langle \phi \rangle^f \equiv \frac{1}{V_f} \int_{V_f} \phi dV$$

such that  $\langle T \rangle^f$  is the intrinsic volume average of the fluid temperature, while  $\langle T \rangle^s$  is the intrinsic volume average of the solid matrix temperature, where  $V_f$  is the volume space which the fluid phase occupies. The porosity  $\varepsilon \equiv V_f/V$  is the volume fraction of the fluid space. The variable  $\phi$  is decomposed into its intrinsic average and the spatial deviation from it:

$$\phi = \langle \phi \rangle^f + \tilde{\phi}$$

Moreover,  $A_{int}$  is the local interfacial area between the fluid and solid matrix, while  $n_i$  is the unit vector pointing outward from the fluid side to solid side. The continuity of both temperature and heat flux is imposed on the interface. Obviously, the parenthetical terms on the right hand-side of Eq. 1 denote the diffusive heat transfer, while the last term describes the interfacial heat transfer between the solid and fluid phases.

One-equation models use the following single equation, which can be reduced by combining Eqs. 1 and 2 under the local thermal equilibrium assumption:

$$\begin{aligned} (\varepsilon \rho_f c_{pf} + (1 - \varepsilon) \rho_s c_s) \frac{\partial \langle T \rangle}{\partial t} + \rho_f c_{pf} \frac{\partial \langle u_j \rangle \langle T \rangle}{\partial x_j} = \frac{\partial}{\partial x_j} \left( (\varepsilon k_f + (1 - \varepsilon) k_s) \frac{\partial \langle T \rangle}{\partial x_j} \right. \\ \left. + \frac{k_f - k_s}{V} \int_{A_{int}} T n_j dA - \varepsilon \rho_f c_{pf} \langle \tilde{u}_j \tilde{T} \rangle^f \right) \end{aligned} \tag{3}$$

where

$$\langle \phi \rangle \equiv \frac{1}{V} \int_V \phi dV$$

is the Darcian average of the variable  $\phi$  such that  $\langle u_j \rangle = \varepsilon \langle u_j \rangle^f$  is the Darcian velocity vector. From the foregoing equation (3), the macroscopic heat flux vector  $q_i = (q_x, q_y, q_z)$  and its corresponding stagnant thermal conductivity  $k_{stag}$  may be defined as follows:

$$q_i = -k_{stag} \frac{\partial \langle T \rangle}{\partial x_i} + \varepsilon \rho_f c_{Pf} \langle \tilde{u}_i \tilde{T} \rangle^f = -(\varepsilon k_f + (1 - \varepsilon) k_s) \frac{\partial \langle T \rangle}{\partial x_i} - (k_f - k_s) \frac{1}{V} \int_{A_{int}} T n_i dA + \varepsilon \rho_f c_{Pf} \langle \tilde{u}_i \tilde{T} \rangle^f \quad (4)$$

Note that the first term in the rightmost expression corresponds to the upper bound of the effective stagnant thermal conductivity based on the parallel model, namely,  $(\varepsilon k_f + (1 - \varepsilon) k_s)$ . Thus, it is the tortuosity term (i.e., the second term) that adjusts the level of the effective stagnant thermal conductivity from its upper bound to a correct one.

Kuwahara et al. (2011) followed Yang and Nakayama (2010) and transformed the energy equations (1) and (2) to the following two-energy equation model along with the effective porosity:

For the fluid phase:

$$\rho_f c_{Pf} \varepsilon \frac{\partial \langle T \rangle^f}{\partial t} + \rho_f c_{Pf} \frac{\partial \langle u_j \rangle \langle T \rangle^f}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \varepsilon^* k_f \frac{\partial \langle T \rangle^f}{\partial x_j} + \varepsilon k_{dis,jk} \frac{\partial \langle T \rangle^f}{\partial x_k} \right) - h_v (\langle T \rangle^f - \langle T \rangle^s) \quad (5)$$

For the solid matrix phase:

$$\rho_s c_s (1 - \varepsilon) \frac{\partial \langle T \rangle^s}{\partial t} = \frac{\partial}{\partial x_j} \left( (1 - \varepsilon^*) k_s \frac{\partial \langle T \rangle^s}{\partial x_j} \right) - h_v (\langle T \rangle^s - \langle T \rangle^f) \quad (6)$$

where the sub- and super-scripts, f and s, refer to fluid and solid phases, respectively. The effective porosity  $\varepsilon^*$  which accounts for the effect of tortuosity on the stagnant thermal conductivity is defined such that the stagnant thermal conductivity is given by

$$k_{stag} = \varepsilon^* k_f + (1 - \varepsilon^*) k_s \quad (7a)$$

namely,

$$\varepsilon^* = \frac{k_s - k_{stag}}{k_s - k_f} = \varepsilon + \frac{\varepsilon k_f + (1 - \varepsilon) k_s - k_{stag}}{k_s - k_f} \quad (7b)$$

such that Eq. 4 gives

$$(\varepsilon^* - \varepsilon) \frac{\partial \langle T \rangle}{\partial x_i} = \frac{1}{V} \int_{A_{int}} T n_i dA \quad (8)$$

As the stagnant thermal conductivity  $k_{stag}$  is given either empirically or theoretically, the effective porosity  $\varepsilon^*$  can easily be evaluated from (7b). Furthermore, the thermal dispersion term is modeled according to the gradient diffusion hypothesis (Nakayama et al. 2006):

$$-\rho_f c_{pf} \left\langle \tilde{u}_j \tilde{T} \right\rangle^f = k_{\text{disk}j} \frac{\partial \langle T \rangle^f}{\partial x_k} \tag{9}$$

while the interfacial heat transfer between the solid and fluid phases is modeled using Newton’s cooling law:

$$\frac{1}{V} \int_{A_{\text{int}}} k_f \frac{\partial T}{\partial x_j} n_j dA = h_v (\langle T \rangle^s - \langle T \rangle^f) \tag{10}$$

where  $h_v$  is the volumetric heat transfer coefficient.

### 3 Physical Model

We shall seek possible exact solutions for convective heat transfer in a tube filled with metal foams, using the foregoing thermal non-equilibrium model. As illustrated in Fig. 1, the air is flowing through an infinitely long tube of diameter  $D$  filled with a metal foam. The tube wall is heated by constant wall heat flux  $q_w$ . As pointed out by Dukhan et al. (2006), the Darcian velocity shows its dependence on the transverse direction only in a small region very close to the wall. Therefore, we may neglect the boundary term (i.e., Brinkman term) and use the slug-flow approximation. Under this approximation, sufficiently away from the entrance, the energy equations (5) and (6) for the individual phases reduce to the following differential equations:

For the fluid phase:

$$\rho_f c_{pf} u_D \frac{\partial \langle T \rangle^f}{\partial x} = (\varepsilon^* k_f + \varepsilon k_{\text{dis},yy}) \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \langle T \rangle^f}{\partial r} \right) - h_v (\langle T \rangle^f - \langle T \rangle^s) \tag{11}$$

For the solid phase:

$$(1 - \varepsilon^*) k_s \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \langle T \rangle^s}{\partial r} \right) - h_v (\langle T \rangle^s - \langle T \rangle^f) = 0 \tag{12}$$

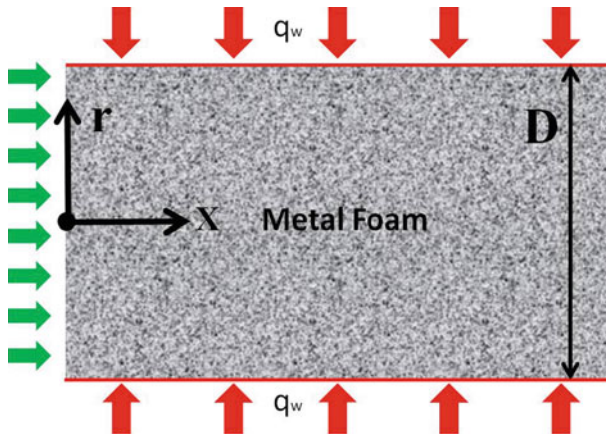


Fig. 1 Heat transfer in a tube filled with a metal foam bounded by constant heat flux wall

with the boundary condition:

$$q_w = (\varepsilon^* k_f + \varepsilon k_{dis,yy}) \left. \frac{\partial \langle T \rangle^f}{\partial r} \right|_{r=D/2} + (1 - \varepsilon^*) k_s \left. \frac{\partial \langle T \rangle^s}{\partial r} \right|_{r=D/2} = \text{const.} \tag{13}$$

### 4 Exact Solutions

Equations 11 and 12 are added to form

$$\rho_f c_{pf} u_D \frac{\partial \langle T \rangle^f}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( (\varepsilon^* k_f + \varepsilon k_{dis,yy}) r \frac{\partial \langle T \rangle^f}{\partial r} + (1 - \varepsilon^*) k_s r \frac{\partial \langle T \rangle^s}{\partial r} \right) \tag{14}$$

where  $u_D = \langle u \rangle$  is the Darcian velocity. Upon integrating the foregoing equation (14) across the tube cross-section with the boundary condition given by Eq. 13, the energy balance readily gives

$$\rho_f c_{pf} u_D \frac{d \langle T \rangle_B^f}{dx} = \frac{4q_w}{D}$$

where  $\langle T \rangle_B^f$  is the bulk temperature of the fluid phase. Hence,

$$\frac{d \langle T \rangle_B^f}{dx} = \frac{\partial \langle T \rangle^f}{\partial x} = \frac{\partial \langle T \rangle^s}{\partial x} = \frac{4q_w}{\rho_f c_{pf} u_D D} \tag{15}$$

which can be substituted into Eq. 14 to give

$$\frac{1}{r} \frac{\partial}{\partial r} \left( (\varepsilon^* k_f + \varepsilon k_{dis,yy}) r \frac{\partial \langle T \rangle^f}{\partial r} + (1 - \varepsilon^*) k_s r \frac{\partial \langle T \rangle^s}{\partial r} \right) = 4 \frac{q_w}{D}$$

This may be integrated as

$$(\varepsilon^* k_f + \varepsilon k_{dis,yy}) \frac{\partial \langle T \rangle^f}{\partial r} + (1 - \varepsilon^*) k_s \frac{\partial \langle T \rangle^s}{\partial r} = 2 \frac{q_w D}{r}$$

where the symmetry condition at  $r = 0$  is exploited. The equation can further be integrated as

$$(\varepsilon^* k_f + \varepsilon k_{dis,yy}) [(T_w - \Delta T) - \langle T \rangle^f] + (1 - \varepsilon^*) k_s (T_w - \langle T \rangle^s) = \frac{q_w}{D} \left( \left( \frac{D}{2} \right)^2 - r^2 \right) \tag{16}$$

where  $\langle T \rangle^s|_{r=D/2} = T_w$  and  $\langle T \rangle^f|_{r=D/2} = T_w - \Delta T$ . The degree of thermal non-equilibrium,  $\Delta T = (\langle T \rangle^s - \langle T \rangle^f)|_{r=\pm D/2}$ , must be prescribed. The foregoing relationship (16) between the solid and fluid temperatures is substituted into Eq. 12 to obtain the following ordinary differential equation in terms of  $(\langle T \rangle^s - T_w)$  as

$$\begin{aligned} (1 - \varepsilon^*) k_s \frac{1}{r} \frac{d}{dr} \left( r \frac{d(\langle T \rangle^s - T_w)}{dr} \right) - \frac{k_{stag} + \varepsilon k_{dis,yy}}{\varepsilon^* k_f + \varepsilon k_{dis,yy}} h_v (\langle T \rangle^s - T_w) \\ = - \frac{h_v q_w}{(\varepsilon^* k_f + \varepsilon k_{dis,yy}) D} r^2 + h_v \left( \Delta T + \frac{q_w D}{4(\varepsilon^* k_f + \varepsilon k_{dis,yy})} \right) \end{aligned} \tag{17}$$

Note that  $(\langle T \rangle^s - T_w)$  is a function of  $r$  alone, since  $dT_w/dx = 4q_w/\rho_f c_{pf} u_D D$ . This ordinary differential equation, after some manipulations, yields

$$\frac{\langle T \rangle^s - T_w}{Dq_w / (k_{stag} + \varepsilon k_{disyy})} = \left( \frac{\varepsilon^* k_f + \varepsilon k_{disyy}}{k_{stag} + \varepsilon k_{disyy}} \frac{\Delta T}{Dq_w / (k_{stag} + \varepsilon k_{disyy})} - \frac{1}{(\lambda D/2)^2} \right) \times \left( \frac{I_0(\lambda r)}{I_0(\lambda D/2)} - 1 \right) + \frac{1}{4} \left( \left( \frac{r}{D/2} \right)^2 - 1 \right) \tag{18}$$

where

$$\lambda = \sqrt{\frac{\left( \frac{k_{stag}}{k_f} + \varepsilon \frac{k_{disyy}}{k_f} \right) \left( \frac{h_v}{k_f} \right)}{\left( \varepsilon^* + \varepsilon \frac{k_{disyy}}{k_f} \right) (1 - \varepsilon^*) \frac{k_s}{k_f}}} \tag{19}$$

and  $I_0$  is the modified zero order Bessel function of the first kind. Equation 18 together with Eq. 16 gives the temperature of the fluid phase:

$$\begin{aligned} & \frac{\langle T \rangle^f - T_w}{Dq_w / (k_{stag} + \varepsilon k_{disyy})} \\ &= \frac{1}{4} \left( \left( \frac{r}{D/2} \right)^2 - 1 \right) + \frac{(1 - \varepsilon^*) k_s}{(\varepsilon^* k_f + \varepsilon k_{disyy}) (\lambda D/2)^2} \left( \frac{I_0(\lambda r)}{I_0(\lambda D/2)} - 1 \right) \\ & - \frac{\Delta T}{Dq_w / (k_{stag} + \varepsilon k_{disyy})} \left( 1 + \frac{(1 - \varepsilon^*) k_s}{k_{stag} + \varepsilon k_{disyy}} \left( \frac{I_0(\lambda r)}{I_0(\lambda D/2)} - 1 \right) \right) \end{aligned} \tag{20}$$

As for the degree of thermal non-equilibrium,  $\Delta T$ , let us consider the two asymptotic conditions, namely, the local thermal equilibrium condition at the wall, i.e.,  $\Delta T = 0$ , and the local uniform heat flux condition at the wall, as given by

$$q_w = \frac{\varepsilon^* k_f + \varepsilon k_{disyy}}{\varepsilon} \frac{\partial \langle T \rangle^f}{\partial r} \Big|_{r=D/2} = \frac{1 - \varepsilon^*}{1 - \varepsilon} k_s \frac{\partial \langle T \rangle^s}{\partial r} \Big|_{r=D/2} \tag{21}$$

which gives

$$\frac{\Delta T}{Dq_w / (k_{stag} + \varepsilon k_{disyy})} = \frac{k_{stag} + \varepsilon k_{disyy}}{\varepsilon^* k_f + \varepsilon k_{disyy}} \left( \frac{(1 - \varepsilon) \frac{k_{stag} + \varepsilon k_{disyy}}{(1 - \varepsilon^*) k_s} - 1}{\frac{D \frac{dI_0(\lambda r)}{dr} \Big|_{r=D/2}}{I_0(\lambda D/2)}} + \frac{1}{(\lambda D/2)^2} \right) \tag{22}$$

Note that

$$\begin{aligned} D \frac{dI_0(\lambda r)}{dr} \Big|_{r=D/2} &= I_1(\lambda D/2) \left( \frac{\lambda D}{2} - \frac{8}{\lambda D} \right) + \frac{\lambda D}{2} I_3(\lambda D/2) \\ &+ 2(I_0(\lambda D/2) + I_2(\lambda D/2)) \end{aligned} \tag{23}$$

where  $I_{1,2,3}$  are the modified Bessel functions of the first, second and third order, respectively.



### 5 Illustrative Temperature Profiles and Nusselt Number

Calmidi and Mahajan (1999, 2000) examined experimental data available for the case of aluminum foam and air combination and proposed the empirical correlations for the stagnant thermal conductivity, the volumetric heat transfer coefficient and the dispersion coefficient as follows:

$$\frac{k_{stag}}{k_f} = \varepsilon + 0.19 (1 - \varepsilon)^{0.763} \sigma \tag{24}$$

$$Nu_v = \frac{h_v d_m^2}{k_f} = 8.72 (1 - \varepsilon)^{1/4} \left( \frac{1 - e^{-(1-\varepsilon)/0.04}}{\varepsilon} \right)^{1/2} \left( \frac{u_D d_m}{\nu} \right)^{1/2} Pr^{0.37} \tag{25}$$

$$\frac{\varepsilon k_{dis,yy}}{k_f} = 0.06 \left( \frac{\rho_f c_{pf} u_D \sqrt{K}}{k_f} \right) \tag{26}$$

The permeability  $K$  is given by the following empirical correlation (Calmidi and Mahajan 1999):

$$K/d_m^2 = 0.00073 (1 - \varepsilon)^{-0.224} \left( \frac{1.18}{1 - e^{-(1-\varepsilon)/0.04}} \sqrt{\frac{1 - \varepsilon}{3\pi}} \right)^{-1.11} \tag{27}$$

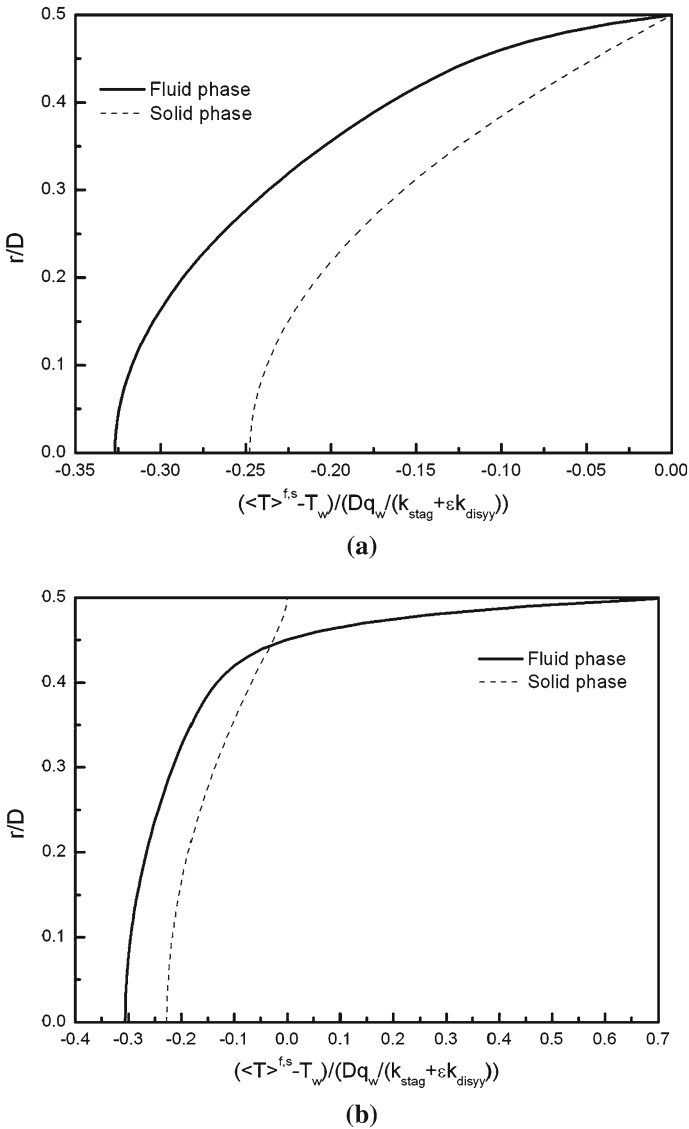
where  $d_m$  is the pore diameter.

Both fluid and solid temperature profiles over the duct cross-section for the case of aluminum-foam and air combination, with  $\sigma = 8200$ ,  $\varepsilon = 0.95$ ,  $\rho_f c_{pf} u_D D / k_f = 5000$ ,  $d_m / D = 0.1$  and  $K / d_m^2 = 0.015$ , are presented in Fig. 2a and b for these two asymptotic wall conditions, namely, the local thermal equilibrium wall condition, i.e.,  $\Delta T = 0$  and the local uniform heat flux wall condition, respectively. Equations 24 to 27 are used to evaluate the stagnant thermal conductivity, the volumetric heat transfer coefficient and the dispersion coefficient. Note that  $k_{stag} / k_f = 160$ ,  $\varepsilon k_{dis,yy} / k_f = 3.67$  and  $\lambda D = 42.1$ .

It should be noted that the local uniform heat flux condition at the wall, as shown in Fig. 2b, leads us to negative  $\Delta T = (\langle T \rangle^s - \langle T \rangle^f)|_{r=D/2}$ , since, under such a condition, the fluid temperature gradient towards the wall becomes so high in order to generate the required uniform heat flux on the adjacent wall. This makes the fluid temperature exceed the solid temperature towards the wall. However, this asymptotic condition may never be realized in practical applications. The reality is much closer to what would be realized under the other asymptotic condition of local thermal equilibrium wall ( $\Delta T = 0$ ), as illustrated in Fig. 2a, since the base materials usually exhibit sufficiently high thermal conductivity.

Both figures clearly show that the solid temperature in the core region is always substantially higher than the fluid temperature for the case of tubes with constant heat flux wall, irrespective of the degree of thermal non-equilibrium  $\Delta T$ . Hence, the local thermal equilibrium assumption fails to be valid for the case of constant heat flux wall.

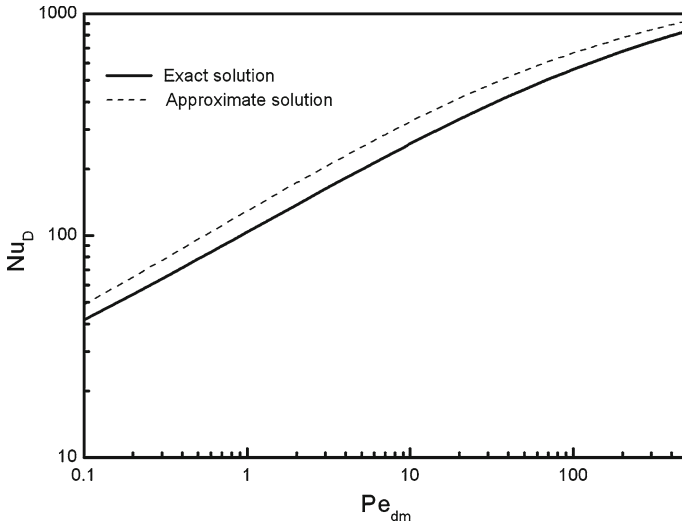
The corresponding Nusselt number may be evaluated from



**Fig. 2** Fluid and solid temperature profiles in a tube filled with a metal foam bounded by constant heat flux wall. **a** Local thermal equilibrium wall ( $\Delta T = 0$ ). **b** Local uniform heat flux wall

$$Nu_D = \frac{q_w D}{(T_w - \langle T \rangle_B^f) k_f} = \frac{k_{stag} + \epsilon k_{disyy}}{k_f} \frac{1}{\frac{8}{D^2} \int_0^{D/2} \left( \frac{T_w - \langle T \rangle^f}{q_w D / (k_{stag} + \epsilon k_{disyy})} \right) r dr} \quad (28)$$

Equation 20 may be substituted into the foregoing equation to find the Nusselt number for the case of local thermal equilibrium at the wall ( $\Delta T = 0$ ), which is presented in Fig. 3. The figure shows a substantial increase in the Nusselt number with the Peclet number based on the pore diameter, due to both high stagnant thermal conductivity and thermal dispersion.



**Fig. 3** Nusselt number for a tube filled with a metal foam bounded by constant heat flux wall

### 6 Approximate Analysis

An approximate treatment based on an integral method is presented here to confirm the validity of the foregoing exact solutions. We shall utilize Eq. 17, which, for the case of local thermal equilibrium wall ( $\Delta T = 0$ ), may be integrated as

$$\eta \frac{d\theta}{d\eta} - \left(\frac{\lambda D}{2}\right)^2 \int_0^\eta \eta \theta d\eta = \frac{1}{4} \left(\frac{\lambda D}{2}\right)^2 \left(\frac{\eta^2}{2} - \frac{\eta^4}{4}\right) \tag{29}$$

where

$$\theta(\eta) = \frac{((T)^s - T_w)(k_{stag} + \epsilon k_{dis,yy})}{q_w D} \tag{30}$$

and

$$\eta = \frac{r}{(D/2)} \tag{31}$$

Further integration leads to

$$-\theta(0) - \left(\frac{\lambda D}{2}\right)^2 \int_0^1 \left(\frac{1}{\eta} \int_0^\eta \eta \theta d\eta\right) d\eta = \frac{3}{64} \left(\frac{\lambda D}{2}\right)^2 \tag{32}$$

The solid phase temperature profile  $\theta(\eta)$  may be assumed as

$$\theta(\eta) = \theta(0) (1 - \eta^2) \tag{33}$$

Substitution of Eq. 33 into Eq. 32 yields

$$\theta(0) = -\frac{1}{4 \left(1 + \frac{16}{3} \left(\frac{2}{\lambda D}\right)^2\right)} \tag{34}$$

From Eq. 16, we find

$$\frac{T_w - \langle T \rangle^f}{Dq_w / (k_{\text{stag}} + \varepsilon k_{\text{dis,yy}})} = \frac{(1 - \varepsilon^*) k_s \theta(\eta) + (k_{\text{stag}} + \varepsilon k_{\text{dis,yy}}) \frac{1}{4} (1 - \eta^2)}{\varepsilon^* k_f + \varepsilon k_{\text{dis,yy}}} \quad (35)$$

Thus, the Nusselt number of our concern may be evaluated according to (28) and (35) as

$$Nu_D = \frac{k_{\text{stag}} + \varepsilon k_{\text{dis,yy}}}{k_f} \left( \frac{8 (\varepsilon^* k_f + \varepsilon k_{\text{dis,yy}})}{k_{\text{stag}} + \varepsilon k_{\text{dis,yy}} - \frac{(1 - \varepsilon^*) k_s}{1 + \frac{16}{3} \left(\frac{2}{\lambda D}\right)^2}} \right) \quad (36)$$

The approximate curve for the Nusselt number based on Eq. 36 is presented in Fig. 3, which shows reasonably good agreement with the exact curve.

## 7 Conclusions

Thermally fully developed flow in a tube filled with a porous medium, subject to constant wall heat flux tube was treated using a local thermal non-equilibrium model. Exact solutions are obtained for the cases of thermally fully developed flow within a constant heat flux tube filled with a porous medium. An aluminum-foam and air combination was considered as an illustrative example. The volume averaged temperature profiles thus obtained for the air and aluminum phases reveal that the aluminum temperature is much higher than the air temperature, suggesting that the local thermal equilibrium assumption may fail for the cases of constant heat flux wall. An approximate solution was also proposed to evaluate the Nusselt number for the case of uniform wall heat flux. The solution agrees fairly well with the exact solution.

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