A General Macroscopic Turbulence Model for Flows in Packed Beds, Channels, Pipes, and Rod Bundles

This study focuses on Nakayama and Kuwahara's two-equation turbulence model (1999, "A Macroscopic Turbulence Model for Flow in a Porous Medium," J. Fluids Eng., 121, pp. 427–433) and its modifications, previously proposed for flows in porous media, on the basis of the volume averaging theory. Nakayama and Kuwahara's model is generalized so that it can be applied to most complex turbulent flows such as cross flows in banks of cylinders and packed beds, and longitudinal flows in channels, pipes, and rod bundles. For generalization, we shall reexamine the extra production terms due to the presence of the porous media, appearing in the transport equations of turbulence kinetic energy and its dissipation rate. In particular, we shall consider the mean flow kinetic energy balance within a pore, so as to seek general expressions for these additional production terms, which are valid for most kinds of porous media morphology. Thus, we establish the macroscopic turbulence model, which does not require any prior microscopic numerical experiments for the structure. Hence, for the given permeability and Forchheimer coefficient, the model can be used for analyzing most complex turbulent flow situations in homogeneous porous media without a detailed morphological information. Preliminary examination of the model made for the cases of packed bed flows and longitudinal flows through pipes and channels reveals its high versatility and performance. [DOI: 10.1115/1.2969461]

Keywords: turbulence model, packed bed, porous media, channel, pipes, volume averaging

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Introduction

Turbulent flows take place in complex geometrical configurations such as in the core of nuclear reactor and in the engine compartments of ground vehicles or aircrafts. However, even with the most powerful computer available today, it is still impractical to perform direct numerical simulation of the turbulent flows within such complex geometrical configurations. An alternative and practical approach may be to appeal to a macroscopic mathematical description designed for turbulent flows in porous media. There are two different paths in the literature to derive macroscopic equations for turbulent flows in porous media. Antheo and Lage [1] obtained a two-equation turbulence model applying the Reynolds averaging operator to the volume-averaged macroscopic equations, namely, Vafai and Tien's equations [2], whereas Nakayama and Kuwahara [3] chose to obtain a two-equation turbulence model by spatially averaging the turbulence transport equations associated with the turbulence kinetic energy, following the procedure taken by Masuoka and Takatsu [4] for their zero-equation model. Pedras and de Lemos [5] showed that the two approaches lead to two different definitions of macroscopic turbulence kinetic energy. In particular, they presented that the definition of the macroscopic turbulence kinetic energy in the latter approach, namely, applying the spatial averaging operator to the Reynolds averaged equations, allows one to take account of the turbulence within a pore, while that in the former approach does not. Extensive discussions on this controversial issue on the two different approaches are available in the literature [6,7].

Within recent years, Nakayama and Kuwahara’s model [3] and its modifications have been extensively used to investigate a variety of turbulent flows, such as flows in a packed bed [8,10], suppressant flows in engine nacelle clutter [11], longitudinal flows in channels, pipes, and rod bundles [11], flows through vegetable stacks [12], flows through densely vegetated channels [13], and flows through a stratified porous medium [14]. This recent record shows that the approach based on the spatial average of the Reynolds average is preferred in most engineering applications.

These macroscopic turbulence transport equations involve additional source terms, which quantify the extra production terms for the turbulence kinetic energy and its dissipation rate due to the presence of the porous media. The source terms vary in formulations, as studied by Guo et al. [8]. They examined three different expressions for the terms, namely, Nakayama and Kuwahara’s model, Pedras and de Lemos [5], and Takeda [15], to study the gas flow through a randomly packed bed and found that three models perform rather differently with Nakayama and Kuwahara’s model giving the most reasonable eddy viscosity. In these models, however, the extra source terms are commonly formulated based on numerical experiment results from two-dimensional periodic structures. Even though reasonable success has been made for the flows in a randomly packed bed with Nakayama and Kuwahara’s model, a more general model, which does not require such a detailed morphology, is in need for practical engineering applications.

In this paper, we shall focus on Nakayama and Kuwahara’s two-equation turbulence model and its modifications and try to generalize them for a universal use in most complex flows in homogeneous porous media. For generalization, we shall reexamine these extra production terms due to the presence of the porous media, appearing in the transport equations of turbulence.
kinetic energy and its dissipation rate, and seek general expressions for these additional production and dissipation terms, valid for most kinds of porous media morphology. In particular, we shall investigate the mean flow kinetic energy balance and model these extra terms accordingly to establish the macroscopic turbulence model, which does not require any prior numerical experiments for the structure, and can be used for most kinds of porous media morphology without detailed morphological information.

Volume-Averaged Equations

Along the lines of the approach proposed by Nakayama [16] and Nakayama and Kuwahara [3], we integrate the set of Reynolds averaged equations, namely, the continuity and the Navier–Stokes equations, along with the standard two-equation model of turbulence, namely, the transport equations of turbulence kinetic energy and its dissipation rate, over an elementary control volume \( V \), which is much larger than a microscopic (pore structure) characteristic size but much smaller than a macroscopic characteristic size. The resulting transport equations for macroscopic momentum, turbulence kinetic energy, and its dissipation rate run as

\[
\frac{\partial \overline{u_j}}{\partial t} + \frac{\partial}{\partial x_j} (\overline{u_i u_j}) = \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial \overline{u}}{\partial x_k} \right) \right] + \frac{1}{V} \int_{A_{int}} \frac{\partial \overline{u_i}}{\partial x_j} dA - \frac{\partial}{\partial x_j} \left( \overline{u_i u_i} - \overline{u} \overline{u_i} \right)
\]

where the underlined terms need to be expressed in terms of determinable variables. The mean strain tensor is given by

\[
s_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

where the Reynolds stress \( -\rho \overline{u_i u_j} \) is modeled according to the effective viscosity formula. Moreover, the symbol

\[
\langle a \rangle^i = \frac{1}{V} \int_V a dV
\]

denotes the intrinsic average of \( a \). For integrations, the spatial averaging rules are exploited as follows:

\[
\langle a_1 a_2 \rangle = \langle a_1 \rangle \langle a_2 \rangle + \langle a_1 \overline{a_2} \rangle^i
\]

\[
\left[ \frac{\partial a}{\partial x_j} \right]^i = \frac{\partial \overline{a}}{\partial x_j} + \frac{1}{V} \int_{A_{int}} a n_i dA
\]

where \( A_{int} \) is the total interface between the fluid and solid phases while \( n_i \) is the unit vector pointing normally outward from the fluid to the solid side. In this section, the single prime is reserved for the turbulence fluctuating quantities, whereas the double prime is used to indicate the deviation from the intrinsic average, such that

\[
a'' = a - \langle a \rangle
\]

The eddy diffusivity \( \nu_e \) is expressed in terms of the intrinsically averaged turbulence kinetic energy \( \langle k \rangle \) and its dissipation rate \( \langle e \rangle \) as

\[
\nu_e = c_D \frac{\langle k \rangle^2}{\langle e \rangle}
\]

where \( c_D = 0.09 \) is the empirical constant.

Comparison of Turbulence Models

In the numerical study of turbulent flow through a periodic array, Kuwahara et al. [17] concluded that Forchheimer-extended Darcy’s law holds even in the turbulent flow regime in porous media. The experimental data, provided by Fand et al. [18] and the recent large eddy simulation (LES) study on a periodic porous structure by Kuwahara et al. [19], also support the validity of Forchheimer-extended Darcy’s law. In Fig. 1, such LES results obtained for cross flows through an array of rods are presented in a dimensionless fashion against the Reynolds number \( Re_f \) on a log-log plot.

In the momentum equation was adopted by Pedrás and de Lemos [5]. The dispersion terms such as \( -\overline{u'' k''/\nu} \) in Eq. (2) and \( -\overline{\nu_e' e'_i} \left( u_i'' \right)^2 \) in Eq. (3) may be modeled according to the gradient hypothesis, as done in Nakayama and Kuwahara [3]. Therefore, in order to close the set of macroscopic equations, we only need to express the extra production terms due to the presence of the porous media, appearing in Eqs. (2) and (3), namely,

\[\text{Transactions of the ASME}\]
Table 1 Expressions for extra production terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>( S₁ = 39 \phi(1 - \phi)^{3/2} \frac{\langle \overline{u} \rangle^2 \langle \overline{u} \rangle^2}{d} )</td>
</tr>
<tr>
<td>S₂</td>
<td>( S₂ = 3.7 \phi^2(1 - \phi)^{3/2} \frac{\langle \overline{u} \rangle^2 \langle \overline{u} \rangle^2}{d} )</td>
</tr>
<tr>
<td>S₃</td>
<td>( S₃ = 411c₂ \phi^{3/2}(1 - \phi)^{3/2} \frac{\langle \overline{u} \rangle^2 \langle \overline{u} \rangle^2}{d} )</td>
</tr>
<tr>
<td>S₄</td>
<td>( S₄ = 0.28 \phi \frac{\langle \overline{u} \rangle^2 \langle \overline{u} \rangle^2}{k} )</td>
</tr>
<tr>
<td>S₅</td>
<td>( S₅ = 0.28c₂ \phi \frac{\langle \overline{u} \rangle^2 \langle \overline{u} \rangle^2}{k} )</td>
</tr>
</tbody>
</table>

where \( d \) is the particle diameter, \( k \) is the permeability, \( m^j \) is the resistance coefficient, and \( c₂ = 0.09, c₁ = 1.44, c₃ = 1.92, c₄ = 1.00, c₅ = 1.30 \).

in terms of the determinable volume-averaged variables.

The expressions proposed for these extra terms by Nakayama and Kuwahara [3], Pedras and de Lemos [5], and Takeda [15] are listed in Table 1. Both Nakayama and Kuwahara [3] and Pedras and de Lemos [5] appealed to microscopic numerical calculations within a unit structure of rod bundles to determine the constants associated with these extra terms. The resulting expressions, however, are claimed to be valid for packed bed flows as well.

Guo et al. [8] chose a flow through a packed bed and carried out numerical simulations using the three turbulence models to compare the effective eddy diffusivity \( \nu \) average across the cross section of the packed bed. They found that the models of Takeda [15] and Pedras and de Lemos [5] generate an eddy diffusivity one to two orders of magnitude higher than the model of Nakayama and Kuwahara [3], which gives a reasonable order of magnitude as compared with the empirical correlation established by Bey and Eigenberger [20]. Guo et al. pointed out that the models of Takeda [15] and Pedras and de Lemos [5] suffer from a deficiency in the source terms, such that the two equations for the transport of turbulence kinetic energy and its dissipation rate reduce to a single equation far downstream. They also noted that with a fixed pore Reynolds number, the effective eddy diffusivity based on Ref. [3] is insensitive to the change in particle size, whereas the other two models show strong dependency on it, namely, a lower eddy diffusivity corresponding to a larger particle size. The dependency on the particle size for a fixed pore Reynolds number contradicts the fact that the ratio of the effective diffusivity to the molecular diffusivity is only a function of porosity and pore Reynolds number, when the turbulence is controlled mainly by a local equilibrium between its production and dissipation within the voids, as in the case of fully developed turbulent flow in homogeneous porous media. The foregoing comparison suggests that a model capable of predicting a reasonable eddy diffusivity must be compatible with the energy budget prevailing within the voids. Thus, we shall consider the mean flow kinetic energy balance within a pore, to seek simple and yet general expressions for the extra source terms for production and dissipation associated with the presence of porous media.

**General Expressions Based on Mean Flow Kinetic Energy Balance**

Appropriate characteristic time scales for the extra source terms have been discussed extensively in recent publications. Pinson et al. [14] modified the time scale based on their friction factor model so as to modify Nakayama and Kuwahara’s model [3] for the study of turbulent flow in a stratified medium. Their model is capable of capturing the dynamic behavior of turbulence kinetic energy. However, it requires a priori microscopic simulation to determine the volume average wake dissipation in advance; hence, its use may not be straightforward. On the other hand, in order to avoid an overestimation of the dissipation of turbulence kinetic energy, Chandresris et al. [11] chose the production time scale proposed by Chen and Kim [21] and Guo et al. [9], which Nakayama and Kuwahara [3] also have indirectly chosen. We shall follow them and adopt their production time scale.

The mean flow kinetic energy transport equation [22] for clear fluid flow through a pore may be given by:

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho \overline{u_i u_i} \right) + \nabla \cdot \left( \rho \overline{u_i u_j} + \frac{2}{3} \overline{u_i u_i} \right) = 2 \nu \nabla^2 \left( \frac{1}{2} \rho \overline{u_i u_i} \right) + 2 \nu \overline{u_i u_i} \overline{u_j u_j}
\]

(13)

The terms on the left-hand side of the foregoing equation represent spatial transport of mean flow kinetic energy \( \overline{u_i u_i} / 2 \), as the divergence theorem implies. Therefore, they cannot influence the overall aspect of the mean flow kinetic energy balance. (Note that they integrate to zero if the boundary is subject to the no-slip condition, and also that they are negligibly small anyway for homogeneous porous media.) Hence, we have:

\[
\left( \frac{1}{2} \rho \overline{u_i u_i} \right) + \frac{2}{3} \overline{u_i u_i} - 2 \nu \overline{u_i u_i} \overline{u_j u_j} = 0
\]

(14)

This practice, namely, forcing the transport terms to be zero, is often introduced in algebraic models of turbulence, so as to remove the need for empirical results for tuning unknown model constants. Expanding the pressure work term and noting the no-slip condition as...
\[ \langle 2 \nu + v \rangle (s_j s_j)^{\nu} + 2 (v + \nu) (s_j t_j)^{\nu} \]

Note that \( \partial (u_i / \bar{u}_i)^{\nu} / \partial x_i = 0 \) for homogeneous porous media and \( (s_j s_j) \ll (s_i s_i) \) for macroscopically fully developed flows in homogeneous porous media. Hence, we may estimate the extra source terms for production and dissipation due to the presence of porous media, using Forcheimer-extended Darcy’s law as

\[ S_k = 2 \nu (s_j s_j)^{\nu} = \frac{1}{\rho} \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} = \frac{1}{\rho} \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} + \frac{1}{\rho} \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} = \frac{1}{\rho} \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} \]

(15)

and

\[ S_k = c_2 \nu (s_j s_j)^{\nu} + \nu V_f \int \frac{\partial \bar{u}_i / \partial x_j}{A_{ua}} = c_2 (2 \nu (s_j s_j)^{\nu}) \]

(17)

where we exploited the relation

\[ \frac{1}{\rho} \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} = \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} = 4 \frac{(\partial (\bar{u}_i)^{\nu})}{pd_i} = 4 \frac{c_{23} \phi (k)}{32k_2 \phi} \]

(19)

noting \( k = 3d / d_k \), as will be presented by Eq. (29) for channel and pipe flows and the Townsend approximation \( \tau / \rho = c_{23} \phi (k) \). Note that we have chosen the production time scale \( (k_1)^{1/3} / S_k \) rather than the decay time scale \( (k_2)^{1/3} / (\epsilon_1)^{1/3} \) for the foregoing extra source terms.

Thus, the final set of the turbulence transport equations runs as

\[ \frac{\partial (k_1)^{1/3}}{\partial t} + \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} = \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} + \frac{(k_2)/(L \epsilon_1)}{L \epsilon_1} \frac{\partial (k_1)^{1/3}}{\partial x_i} + 2 \nu (s_j s_j)^{\nu} - (\epsilon_1)^{1/3} \phi \]

(18)

and

\[ \frac{\partial (\epsilon_1)^{1/3}}{\partial t} + \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} = \frac{\partial (\bar{u}_i)^{\nu}}{\partial x_i} + \frac{(k_2)/(L \epsilon_1)}{L \epsilon_1} \frac{\partial (\epsilon_1)^{1/3}}{\partial x_i} + 2 \nu (s_j s_j)^{\nu} - (\epsilon_1)^{1/3} \phi \]

(19)

where \( L_{k2} \) and \( L_{k3} \) are the Lewis numbers for mechanical dispersion, which are believed to be close to unity. The empirical and theoretical expressions for thermal dispersion tensor \( (k_{23})_{ij} \) may be found elsewhere \( [24, 25] \). The resulting turbulence model is so general that it can be used for most kinds of porous media morphology, since it only requires the permeability \( K \) and the Forcheimer constant \( b \), which can easily be determined from the pressure drop measurements.

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**Turbulent Flow in a Packed Bed**

For the case of packed bed, the permeability \( K \) and the Forcheimer constant \( b \) may be given by the empirical Ergun equation \( [26] \):

\[ K = \frac{\phi^3}{150(1 - \phi)^2} \]

(22)

and

\[ b = \frac{1.75(1 - \phi)}{\phi d} \]

(23)

Substituting the foregoing expressions in the extra production terms, we have

\[ S_k = \phi^3 b (\bar{u}_i)^{\nu} / (\bar{u}_i)^{\nu} = \frac{1.75(1 - \phi)}{\phi} (\bar{u}_i)^{\nu} (\bar{u}_i)^{\nu} / d^2 \]

(24)

and

\[ S_k = \phi^3 b (\bar{u}_i)^{\nu} (\bar{u}_i)^{\nu} = 4.55c_2 \frac{(1 - \phi)}{\phi} (\bar{u}_i)^{\nu} (\bar{u}_i)^{\nu} / d^2 \]

(25)

The result Eqs. (20) and (21) are consistent with Nakayama and Kuwahara’s expressions \( [3] \), as listed in Table I. In particular, the source terms \( S_k \) and \( S_k \) given by Eqs. (24) and (25) conform to those of Nakayama and Kuwahara \( [3] \) with the coefficients \( \phi^3 b = 1.75(1 - \phi) \) and \( c_2 \phi^3 b = 4.55c_2(1 - \phi) / \phi^2 d^2 \), respectively, in their expressions. This could be partly the reason why their model gave the most reasonable eddy diffusivity for packed bed flows.

The present expressions for \( S_k d (\bar{u}_i)^{\nu} (\bar{u}_i)^{\nu} / d^2 = 1.75(1 - \phi) / \phi \) and \( S_k d^2 c_2(1 - \phi) / \phi^3 d^2 \) are compared in Figs. 2 and 3, respectively, against Nakayama and Kuwahara’s expressions \( [3] \) based on microscopic numerical results from periodic structures for a wide porosity range 0.4 < \( \phi \leq 0.8 \). Despite the
For the case of unidirectional flow without macroscopic mean shear, Eqs. (20) and (21) far downstream readily give

$$\langle \varepsilon \rangle = \phi^3 (\langle \tilde{u} \rangle)^3 = \frac{1.75 (1 - \phi) (\langle \tilde{u} \rangle)^3}{d}$$  \hspace{1cm} (26)

and

$$\langle k \rangle = \phi^2 b \sqrt{\frac{2K}{c_D \phi}} (\langle \tilde{u} \rangle)^2 = 1.75 \sqrt{\frac{2}{150c_D}} (\langle \tilde{u} \rangle)^2 = 0.673 (\langle \tilde{u} \rangle)^2$$  \hspace{1cm} (27)

Hence,

$$\nu = c_D \frac{\langle \tilde{u} \rangle^2}{\nu(e)} = 0.0233 \frac{\phi}{1 - \phi} \frac{\langle \tilde{u} \rangle^2}{\nu}$$  \hspace{1cm} (28)

It is interesting to note that, for homogeneous porous media, the ratio of the limiting value of the volume-averaged kinetic energy to the square of the intrinsically volume-averaged velocity is constant irrespective of the porosity. This constancy is confirmed in Fig. 4 in which the numerical results obtained by Nakayama and Kuwahara [3] for two-dimensional periodic structures appear to stay nearly at the level indicated by Eq. (27) over a wide range of porosity.

In Fig. 5, the effective eddy kinematic viscosity, as given by Eq. (28), is compared against the empirical correlation established by Bey and Eigenberger [20] and also the results obtained by Guo et al. [8] in the Nakayama and Kuwahara model which in the present paper is assumed to be $\phi=0.4$. Guo et al. [8] carried out the computations using Pedras and de Lemos [5] and Takeda [15] and found that their effective viscosities are one to two orders of magnitude higher than the model of the empirical correlation and concluded that Nakayama and Kuwahara’s model gives most reasonable effective viscosity. Although the present model and the Nakayama and Kuwahara model perform better than the other two models, a substantial difference still exists between the present prediction and the empirical correlation proposed by Bey and Eigenberger. Guo et al. [9] listed several reasons associated with experimental uncertainties. Even though experimental error may not be negligible, there is certainly a further need for experimental and theoretical work to improve the macroscopic models.

Nakayama and Kuwahara [3] carried out microscopic computations using the standard two-equation turbulence model for the case of turbulent flow through an array of rods with the Reynolds number based on the Darcian velocity the and length of structural unit $\frac{u_d^2}{v}$. The porosity $\phi=0.75$, and the inlet values of turbulence quantities $\langle \varepsilon \rangle^2/\nu = 10$ and $\langle k \rangle^2/\nu = 30$. The microscopic results were integrated over a unit to obtain the volume-averaged turbulence quantities, which were then compared with those based on their macroscopic model. In Figs. 6(a) and 6(b), these two sets of results for turbulence kinetic energy and its dissipation rate are compared against the macroscopic results obtained using the present macroscopic model, namely, Eqs. (20) and (21). Excellent agreement is seen among the three sets of the results.

### Turbulent Flow in Channels and Pipes

We shall adapt this general model to longitudinal turbulent flows in channels, pipes, and rod bundles such as found in nuclear power circuits and core. For the case of longitudinal flows in channels, pipes, and rod bundles, the permeability $K$ and the Forchheimer constant $b$ may be set according to the Hargrave-Poiseuille flow solution and the Blasius friction law, namely,

$$K = \frac{\phi}{32} d_h^2$$  \hspace{1cm} (29)

such that

$$\frac{1}{\rho} \frac{\partial (\rho u)}{\partial x} = \frac{\phi}{K} (\langle \tilde{u} \rangle)^2 = \frac{32}{d_h} (\langle \tilde{u} \rangle)^2$$  \hspace{1cm} (30)

and

$$\nu = c_D \frac{\langle \tilde{u} \rangle^2}{\nu(e)} = 0.0233 \frac{\phi}{1 - \phi} \frac{\langle \tilde{u} \rangle^2}{\nu}$$  \hspace{1cm} (28)
such that
\[
\frac{1}{\rho} \frac{d \overline{\rho}'}{dx} = \phi^2 \frac{d (\overline{\rho}')^2}{dx} = \frac{\lambda_f}{2d_h} (\overline{\rho}')^2
\]
(32)

where \(d_h\) is the hydraulic diameter of the passage under consideration and \(\lambda_f\) = \(2(2d_h/\rho((\overline{\rho}')^2))d(\overline{\rho}')/dx\) is the Moody friction factor.

For the case of longitudinal flow without macroscopic mean shear such as found in the case of channel, pipe, and rod bundles, Eqs. (20) and (21) far downstream readily give
\[
\langle e \rangle' = \phi^2 b ((\overline{\rho}')^3) = \frac{\lambda_f}{2d_h} ((\overline{\rho}')^3) = \frac{0.158}{((\overline{\rho}')^3)} \frac{d (\overline{\rho}')}{dx} = \frac{0.132}{((\overline{\rho}')^3)} \frac{d (\overline{\rho}')}{dx}
\]
(33)

and
\[
\langle k \rangle' = \phi^2 b \sqrt{\frac{2K}{c_D \phi}} ((\overline{\rho}')^2) = \frac{\lambda_f}{8c_D} ((\overline{\rho}')^2) = \frac{0.00909}{((\overline{\rho}')^3/\nu)^{3/4}}
\]
(34)

Hence,
\[
\frac{\nu}{\nu} = \frac{\langle k \rangle^2}{\nu (\langle \epsilon \rangle')} = \frac{\lambda_f}{32} \frac{((\overline{\rho}')^2)}{\nu} = 0.00909 \left( \frac{((\overline{\rho}')^2/\nu)^{3/4} \right)
\]
(35)

The limiting values of the turbulence kinetic energy obtained for the fully developed channel and pipe flows are plotted in Fig. 7 in terms of the dimensionless form \(\langle k \rangle'((\overline{\rho}')^2)/\nu^{1/4}/((\overline{\rho}')^2)^{1/2}\)
along with the experimental data of Comte-Bellot [27] and Perry et al. [28]. A reasonably good agreement is seen among the values in the figure. Furthermore, it is interesting to note that the present model estimates the value \(c_i = ((\overline{\rho}')^2)/(4(\overline{k}')((\overline{\rho}')^2))\) (focused by Chandresis et al. [11]) to be \(c_i = \sqrt{c_D} = 0.3\) irrespective of the Reynolds number. The constancy of \(c_i\) is clearly confirmed in Fig. 8, which presents the numerical results for channels and pipe flows, obtained by Chandresis et al. [11] using a modified version of Nakayama and Kuwahara’s turbulence model.

Chandresis et al. [11] studied the decay of turbulence inside a stratified medium made by plane channels from both microscopic and macroscopic views. Their macroscopic turbulence model uses the kinetic energy balance, an estimation of the direct viscous dissipation and an ad hoc length scale. The volume-averaged turbulence quantities such as turbulence kinetic energy and its dissipation rate obtained by them for the case of \(\langle \overline{\rho}' \rangle' d_h/\nu = 10^5\) are plotted in Figs. 9(a) and 9(b) along with the macroscopic results obtained by solving Eqs. (20) and (21) and for a uniform unidirectional flow without mean shear with \(\langle k \rangle' = 10(\overline{k}')_{\infty}\) and \(\langle \epsilon \rangle'\) = 15(\overline{\epsilon})_{\infty}\), where \(\langle k \rangle'_{\infty}\) and \(\langle \epsilon \rangle'_{\infty}\) are the fully developed values of turbulence kinetic energy and its dissipation rate. A good agreement among the three sets of the results gives strong support to the present general turbulence model.

Conclusions

Nakayama and Kuwahara’s two-equation turbulence model previously introduced for turbulent flows in porous media has been generalized to treat most kinds of complex turbulent flows in homogeneous porous media. In order to generalize the model, the extra production terms due to the presence of the porous media, appearing in the transport equations of turbulence kinetic energy and its dissipation rate, were re-examined in consideration of the mean flow kinetic energy balance. The resulting turbulence model is so versatile that it can be used to attack cross flows in banks of cylinders and packed beds, and longitudinal flows in channels, pipes, and rod bundles. The model does not require any prior numerical experiments for the structure. Preliminary examination of the model made for the cases of packed bed flows and longi-

![Fig. 7 Effect of Reynolds number on turbulence kinetic energy in channel and pipe flows](image1)

![Fig. 8 Effect of Reynolds number on \(\overline{k}' = ((\epsilon)'d_h)/(4(\overline{k}')((\overline{\rho}')^2))\) in channel and pipe flows](image2)

![Fig. 9 Decay of turbulence in flow through a stratified medium made by channels; (a) turbulence kinetic energy and (b) dissipation rate of turbulence kinetic energy](image3)
tudinal flows through pipes and channels reveals that, for a given permeability and Forchheimer coefficient, it can be used for most kinds of porous media morphology without detailed morphologi-

Nomenclature

\[ A = \text{surface area} \]
\[ A_{\text{int}} = \text{total interface between the fluid and solid} \]
\[ b = \text{Forchheimer constant} \]
\[ c_1, c_2, c_D = \text{turbulence model constants} \]
\[ d = \text{particle diameter} \]
\[ d_h = \text{hydraulic diameter} \]
\[ k = \text{turbulence kinetic energy} \]
\[ K = \text{permeability} \]
\[ L_e = \text{Lewis number} \]
\[ p = \text{pressure} \]
\[ Re = \text{Reynolds number} \]
\[ u_i = \text{velocity vector} \]
\[ x_i = \text{Cartesian coordinates} \]
\[ e = \text{dispersion rate of turbulence kinetic energy} \]
\[ \lambda_f = \text{friction factor} \]
\[ \nu = \text{kinematic viscosity} \]
\[ \nu_l = \text{eddy diffusivity} \]
\[ \sigma_{x_i} = \text{effective Prandtl number} \]
\[ \tau = \text{shear stress} \]
\[ \rho = \text{density} \]
\[ \phi = \text{porosity} \]

Special Symbols

\[ \bar{a} = \text{ensemble mean} \]
\[ a' = \text{turbulent fluctuation} \]
\[ a^* = \text{deviation from intrinsic average} \]
\[ \langle a \rangle = \text{intrinsic average} \]

Subscripts and Superscripts

\[ \text{dis} = \text{dispersion} \]
\[ f = \text{fluid} \]
\[ \text{tor} = \text{tortuosity} \]

References


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#1 Au: ABSTRACT MUST BE SELF-CONTAINED. PLEASE CHECK OUR INSERTIONS

#2 Au: PLEASE SUPPLY DEFINITION OF "LES" IF POSSIBLE

#3 Au: PLEASE SUPPLY LOCATION FOR CONFERENCE IN REF. 10. Please supply first initial for author “Gritzo” in Ref. 10.