A Lumped Parameter Heat Transfer Analysis for Composting Processes With Aeration

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Introduction

Composting is the biological decomposition and stabilization of organic substances. Heat is generated biologically to produce a final product that is stable, free of pathogens and weed seeds, which can be beneficially applied to land. As pointed out by Haug [1] and Li and Jenkins [2], composting is an ancient art, yet engineering that is still often conducted using a “handbook approach.” However, such an approach lacks the knowledge to control various factors involved to achieve the desired end product and economics. A typical composting system with aeration is shown in Fig. 1, where the air is ventilated from the bottom to accelerate the biological processes. The air carries sensible and latent heat away as passing through the matrix, while it is essential to maintain organic decomposition leading to biological heat generation. Thus, the control of aeration requires heat transfer analysis if we are to maintain the optimum temperature for the composting system (which, according to Nakasaki et al. [3], coincides with the optimum reaction temperature of around 60°C). Mathematical modeling of composting processes in such a composting system is still in its infancy, although several attempts [3–8] have been made to simulate the composting reactions.

The authors [9] have recently introduced the volume-averaging theory previously established for the study of porous media with heat generation (e.g., Nakayama et al. [10]). We extended it to establish a complete set of the volume-averaged governing equations appropriate for the analysis of composting processes. As a first step towards our strategic efforts to establish a complete numerical prediction tool for composting operations, we propose a simple lumped parameter model, which can be obtained by integrating the foregoing governing equations.

Heat Balance Equation

The composting pile consists of solid, liquid, and gas. The solid phase includes biodegradable substrates, microbes, and humic substances converted from dead organisms and minerals, whereas the main contents of the gas phase are oxygen, nitrogen, carbon dioxide, and water vapor. Both substrates and microbes are of multi-components. However, we shall consider a composting system of one defined substrate and one defined microbe species. For simplicity and definiteness, let the gas phase (denoted by the super- or subscripts f) refer to the mixture of the gases and water vapor, and let the other, namely, the porous matrix (denoted by the super- and subscripts s), refer to the liquid water, biodegradable substrate, microbes, and uncompostable substances, all of which are assumed to be in thermal equilibrium within the matrix.

Heat is generated due to biological reactions, and transfers from the solid to fluid or vice versa. Under such a non-equilibrium condition, the two-energy equation model [9] may be used, in which two distinctive intrinsic average temperatures, namely, one for the gas f(T) and the porous matrix s(T), are introduced as follows

\[ \rho c_v \frac{dT}{dt} + (u_j) \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \epsilon k_{fj} \frac{\partial T}{\partial x_j} \right) + \frac{1}{V} \int_{A_{sub}} k_f \frac{dT}{dx_j} dA 
\]

(1)

\[ (1 - \varepsilon) \left( \langle W \rangle c_w + \langle S \rangle c_{sub} + \langle X \rangle c_x \right) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j} \left( (1 - \varepsilon) k_{se} \frac{\partial T}{\partial x_j} \right) - \frac{1}{V} \int_{A_{sub}} k_f \frac{dT}{dx_j} dA 
+ (1 - \varepsilon) \left[ H_w \frac{\partial \langle W \rangle}{\partial t} - H_{sub} \frac{\partial \langle S \rangle}{\partial t} + \frac{\partial \langle X \rangle}{\partial t} \right] 
\]

(2)

The last term on the right-hand-side of (2) describes the net volumetric heat generation due to biological reactions. However, Nakayama et al. [10] analytically showed the difference in the two temperatures is quite small as long as the macroscopic characteristic length scale (i.e., the size of the static pile) is much greater than \( \sqrt{\frac{k_f}{h_w}} \) where \( h_w \) is the volumetric heat transfer coefficient. Thus, we may combine the two energy equations, under the local thermal equilibrium (i.e., \( T_s = \langle T \rangle_f = \langle T \rangle \)), to obtain a single energy equation for the composting system

\[ [(1 - \varepsilon) \left( \langle W \rangle c_w + \langle S \rangle c_{sub} + \langle X \rangle c_x \right) + \langle U \rangle c_x] \frac{\partial T}{\partial t} + \varepsilon \rho c_f \frac{\partial T}{\partial t} + \rho c_f c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j} \left[ (1 - \varepsilon) k_{se} \frac{\partial T}{\partial x_j} \right] + (1 - \varepsilon) \left[ H_w \frac{\partial \langle W \rangle}{\partial t} - H_{sub} \frac{\partial \langle S \rangle}{\partial t} + \frac{\partial \langle X \rangle}{\partial t} \right] 
\]

(3)

We integrate the foregoing heat balance equation over the entire composting matrix to obtain the following ordinary differential equation for a lumped parameter analysis

\[ m_e \frac{dT}{dt} = \rho_{sub} c_{sub} \bar{V}_{sub} (T_a - T) + h_A (T_a - T) + H_w \frac{dm_w}{dt} - H_{sub} \frac{dm_{sub} + m_s}{dt} 
\]

(4)

where \( T (\degree C) \) here is the temperature averaged over the entire composting matrix. Note...
where $m = m_w + m_{sub} + m_c + m_u$ is the total mass of the composting matrix, while its specific heat capacity is given by

$$c_p = \frac{m_c c_w + m_{sub} c_{sub} + m_c c_{sub} + m_c c_{sub}}{m_c}$$

where $c_{sub} = c_{c} = c_{i}$ is assumed. We may replace the properties of the gas phase by those of air. The first term on the right-hand side in Eq. (4) denotes the heat carried away by the air supplied at the volume flow rate of $V_{air}$. While the second term represents the heat loss to the surroundings of the temperature $T_a$. Whether or not the lumped parameter model can model the composting process to a quantifiable degree depends on the uniformity of the temperature and concentration distributions within the pile. Since the aeration makes the distributions uniform, the lumped parameter model of this kind is believed to give quantitative information for the average values in the aerated system.

Microbial Growth Rate Equation and Water Transport Equation

We may model the microbial growth rate according to Contois [11] as

$$\frac{dm_w}{dt} = -W_{sub} \frac{dm_{sub}}{dt} = \mu \left( m_w - m_{sub} \right)$$

where $\mu$ is the maximum specific growth rate, which is given by

$$\mu = \begin{cases} \frac{m_w - w_a}{m_w - w_a + \alpha(T)} : & w_a \leq m_w \leq w_1 \\ \frac{w_1 - w_a}{w_1 - w_a + \alpha(T)} : & w_1 \leq m_w \leq w_2 \\ 0 & : \text{otherwise} \end{cases}$$

where $\alpha(T) = \begin{cases} \exp \left[ \frac{1}{T + 273} - \frac{1}{T_M + 273} \right] & : T \leq T_M \\ \frac{T_L - T}{T_L - T_M} & : T_M \leq T \leq T_L \\ 0 & : T_L \leq T \end{cases}$

The coefficient $\alpha$ accounts for the empirical evidence found by Nakasaki et al. [3]. Thus, $\mu$ increases with the temperature following Arrhenius expression up to the optimum reaction temperature $T_M = 60^\circ C$, but decreases for any further temperature rise, as microbial activities are held back. Fujita [8] on the basis of the experiments using the mixtures consisting of dog foods, sludge, and sawdust, recommends the values as follows: $K_a = 0.04, w_a = 0.15, w_1 = 0.6, w_2 = 0.8$, $T_M = 60^\circ C$, $T_L = 80^\circ C$, $E_A = 29,000 J/mol$, $R_A = 8.314 J/mol K$.

The Contois constant $k_c$ is related to the growth yield as $k_c Y = 4 \sim 20$. Here, we fix $k_c = 24$, $Y = 0.5$, and vary $\mu_0$ around a typical value $0.18/h$ (i.e., $5 \times 10^{-5}/s$), which depends on the degradability of the composting material.

In order to close the model, we introduce the water vaporization relationship proposed by Fujita [8], namely

$$\frac{dm_w}{dt} = -W_{sat} \frac{dm_{sub}}{dt} $$

where the saturated vapor concentration is given by

$$W_{sat}(T) = 0.804 \left( \frac{11.96 - \frac{3994}{T + 233.9}}{1 - \exp \left( \frac{11.96 - \frac{3994}{T + 233.9}}{903} \right)} \right)$$

Normalization and Dimensionless Numbers

In order to normalize the water transport equation (10), microbial growth rate equation (7), and heat balance equation (4), we define dimensionless variables of the form

$$t' = \frac{t}{m_0 c_x T_0}, \quad T' = \frac{T - T_a}{T_M - T_a}, \quad m_* = \frac{m}{m_0}, \quad m_{sub} = \frac{m_{sub}}{m_0}, \quad m_c = \frac{m_c}{m_0}$$

and

$$m_{sub} = \frac{m_{sub}}{m_0}, \quad m_{sub} = \frac{m_{sub}}{m_0}, \quad m_0 = \frac{m_0}{m_0}$$

The subscript “0” refers to its initial value. The convection time scale $t_c = m_0 c_x h_0 / \rho a c_x V_{air}$ is used to normalize the time, while the difference between the optimum reaction temperature and ambient temperature is introduced to scale the system temperature. Using these dimensionless variables, the governing equations are normalized as follows

$$\frac{dm_w}{dt'} = -F_w(T'), \quad \frac{dm_{sub}}{dt'} = -\frac{m_*}{1 - \left( m_{sub0} - m_{sub} \right)_1 - \left( m_{sub0} - m_{sub} \right)_2}$$

$$\frac{dm_*}{dt'} = -D a F_{sub}(T)', \quad \frac{dm_*}{dt'} = -D a F_{sub}(T)', \quad m_* = m_{sub0} + Y (m_{sub0} - m_{sub})$$

and

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\[
\left( 1 - \frac{c_w}{c_{w0}} (m_{w0}^* - m_w^*) - \frac{c_{m0}}{c_{m0}} (m_{m0}^* - m_m^*) (1 - Y) \right) \frac{dT}{dt} = -(1 + St)T^* + \frac{H_w}{c_{w0} (T_m - T_d)} F_w(T^*) \\
+ Da \left( \frac{H_{m0}}{c_{m0} (T_m - T_d)} \right) F_{m0}(T^*, m_m^*) (1 - Y) \\
+ \frac{m_{m0}^* m_w^* + Y (m_{m0} - m_m^*)}{m_m^* + Y (m_{m0} - m_m^*) + (m_m^* / k_c)} \\
\times \left[ 1 - (m_{w0}^* - m_w^*) - (m_{m0}^* - m_m^*) (1 - Y) \right]
\]

(16)

where

\[
F_w(T^*) = \frac{c_{w0} W_{w0}(T)}{\rho_{w0} c_{w0} V_{w0}} \exp \left[ \frac{E_A}{R_A} \left( \frac{1}{T_a + 273} - \frac{1}{T_m + 273} \right) \right]
\]

(17a)

\[
F_{m0}(T^*, m_m^*) = \left( \frac{\mu_b}{\rho_{m0} c_{m0} V_{m0}} \right) \exp \left[ \frac{E_A}{R_A} \left( \frac{1}{T_a + 273} - \frac{1}{T_m + 273} \right) \right]
\]

(17b)

The Stanton number accounts for the heat loss to the environment while the Damkohler number corresponding with the ratio of reaction rate to convection rate controls the biological heat generation within the matrix. Alternatively, the Damkohler number may be interpreted as the ratio of the convection time scale \( t_{conv} = m_{w0}^* c_w / \rho_{w0} c_{w0} V_{w0} \) to that of the microbial reaction. As we give \( St, Da, T_a \) (or \( c_{m0} (T_m - T_d) / H_w \)) and the initial values \( m_{w0}, m_{m0}, m_{m0}^*, \) and \( T = 0 \), we may readily integrate the foregoing ordinary differential equations to determine \( T^*(t), m_w^*(t), \) and \( m_{m0}^*(t) \), and subsequently find \( m_1^*(t) \) and \( m_1(t) \) from \( m_{w0}^* + Y (m_{m0} - m_m^*) + (m_m^* / k_c) \).

Results and Discussion

As a reference case, we consider a composting process within the same static pile as investigated by Fujita [8], in which the air continuously flows into the static pile from the bottom by forced ventilation and the exhaust gas escapes from the top exposed to the ambient air at the rate of \( \dot{V}_a = 0.262 \text{ m}^3 / \text{s} \). The volume of the static pile is 235.5 m\(^3\) and the initial total mass \( m_{w0} = 47.100 \text{ kg} \) such that the apparent density is 200 kg/m\(^3\). Fujita [8] experimentally evaluated the heat transfer coefficients at the inner and outer walls in the range from 9 to 11 W/m\(^2\)/K. The overall thermal conductance through the concrete wall was estimated to be \( h_A = 0.640 \text{ W/K} \). The initial masses are given by \( m_{w0} = 28.260 \text{ kg}, m_{m0} = 14.130 \text{ kg}, \) and \( m_{m0}^* = 471 \text{ kg} \) such that \( m_{w0}^* = 0.6, m_{m0} = 0.3, \) and \( m_{m0}^* = 0.01 \). The physical properties used in computations are as follows: \( \rho_{w0} = 1200 \text{ kg/m}^3, c_w = 1400 \text{ J/kg K}, c_w = 4200 \text{ J/kg K}, c_{m0} = 2100 \text{ J/kg K}, H_w = 2.44 \times 10^5 \text{ J/kg K}, \) and \( H_{m0} = 1.76 \times 10^7 \text{ J/kg K} \), such that \( c_{m0} = m_{w0}^* c_w + (1 - m_{m0}) c_{m0} = 3360 \text{ J/kg K} \). and \( t_{ref} = \rho_{w0} c_{w0} \dot{V}_a / m_{w0} c_w = 100 \text{ h} \) (i.e., \( 3.6 \times 10^5 \text{ s} \)). For this reference case, we set \( \mu_b = 0.18 / \text{h} \) (i.e., \( 5 \times 10^{-5} / \text{s} \)) and \( T_n = 20^\circ \text{C} \), such that

**Fig. 2** Effects of \( \mu_b \) on system temperature

\[
St = \left( \frac{hA m_{w0}}{\rho_{w0} c_{w0} V_{w0}} \right) = 1.48
\]

(19a)

\[
Da = \frac{c_{m0} \mu_b}{\rho_{m0} c_{m0} V_{m0}} \exp \left[ \frac{E_A}{R_A} \left( \frac{1}{T_a + 273} - \frac{1}{T_m + 273} \right) \right]
\]

(19b)

**Fig. 3** Effects of \( \mu_b \) on total mass

**Fig. 4** Effects of \( \mu_b \) on system temperature

**Fig. 5** Effects of \( \mu_b \) on system temperature
system temperature and a longer time to attain its peak temperature. Naturally, Fig. 3 shows that the total mass decreases more slowly for a smaller $\mu_b$. Especially when $\mu_b$ as small as 0.08, the system temperature never exceeds the optimum reaction temperature of 60°C.

The effects of ambient temperature $T_a$ on the system are similar to those of $\mu_b$. We note from Eq. (19b) that any decrease in either $\mu_b$ or $T_a$ results in lowering the value of $D_a$. As will be shown later using the dimensionless presentation, it is the Damkohler number $D_a$ that virtually controls the time for the system to reach its peak temperature.

**Effects of the Thermal Conductance (St) on the Composting Process.** The size of the static pile directly reflects on the value of $\frac{hA}{m_{0}}$, which naturally increases for a smaller size. Fujita [8] gives $hA/m_{0}=0.0136$ W/kg K for $m_{0}=47,100$ kg (i.e., the reference case) and $hA/m_{0}=0.0445$ W/kg K for $m_{0}=2.51$ kg. It is assumed that the air is supplied in such a fashion that the volume flow rate per unit mass of the matrix is held constant, say $V_{air}/m_{0}=5.56 \times 10^{-6}$ m$^3$/kg s (i.e., 0.02 m$^3$/kg h). This practice helps attain optimum microbial activities, without carrying much heat away from the system. Thus, for the same ambient conditions, a decrease in the size of the static pile results in an increase in $St$ (see Eq. (19a)), while $D_a$ (see Eq. (19b)) stays constant. Computations were made for the cases ranging from $hA/m_{0}=0.0136$ W/K kg to 0.0667 W/K kg ($St=1.48$ to 7.14) and the results are shown in Fig. 4. The figure clearly shows that heat loss shortens the plateau of high temperature period; and decelerates consumption of the mass in the system. It is essential to design the system such that $St$ stays sufficiently small (say $St<5$) for the system temperature to exceed the optimum reaction temperature.

**Dimensionless Presentation.** In addition to the reference case, two distinctive cases; namely $(T_a, \mu_b)=(10°C, 0.247/h)$ and $(0°C, 0.43/h)$, in which $D_a$ remains the same value $D_a=0.359$ as the reference case $(T_a, \mu_b)=(20°C, 0.18/h)$, are considered to investigate possible similarity in the results for fixed $D_a$.

The temperature results for three cases are presented in dimensional and dimensionless forms in Figs. 5 and 6, respectively, while the temporal development of the total mass is plotted in Fig. 7 using a dimensionless form. Figure 6 clearly shows that all three normalized curves overlap one another in a considerably wide range, from the beginning of the temperature rise to the end of the temperature plateau. (After the plateau, the temperature drops faster for the lower ambient temperature.)

From both Figs. 6 and 7, we may conclude that, for a given Stanton number St, the Damkohler number $D_a$ virtually controls the system. Thus, it should be taken as one of the important parameters for designing the composting process with aeration.
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Nomenclature

\[\begin{align*}
A &= \text{total surface area of the static pile (m}^2) \\
c &= \text{heat capacity (J/kg k)} \\
Da &= \text{Damkohler number} \\
E_a &= \text{activation energy (J/mol)} \\
h &= \text{overall heat transfer coefficient (W/m}^2\text{K)} \\
H_w &= \text{latent heat of water vaporization (J/kg)} \\
H_{\text{sub}} &= \text{latent heat of metabolic reaction (J/kg)} \\
k_e &= \text{Contios constant} \\
k_{fe} &= \text{effective thermal conductivity of fluid (W/m K)} \\
k_{fe} &= \text{effective thermal conductivity of solid (W/m K)} \\
m &= \text{mass (kg)} \\
u &= \text{Darciian velocity (m/s)} \\
R_g &= \text{universal gas constant (J/mol k)} \\
\langle S \rangle_f &= \text{volume average concentration of substrate (kg/m}^3\rangle \\
\text{St} &= \text{Stanton number} \\
\tau &= \text{time (s)} \\
T &= \text{system temperature (K)} \\
T_a &= \text{ambient temperature (K)} \\
T_M &= \text{optimum reaction temperature (K)} \\
\langle U \rangle_f &= \text{volume average concentration of uncompostable (kg/m}^3\rangle \\
V_{\text{air}} &= \text{volume flow rate of air (m}^3\text{/s)} \\
\langle W \rangle_f &= \text{volume average concentration of water (kg/m}^3\rangle \\
w_{l,1,2} &= \text{constants associated with the maximum specific growth rate} \\
\langle X \rangle_f &= \text{volume average concentration of microbes (kg/m}^3\rangle \\
Y &= \text{growth yield} \\
\alpha &= \text{function associated with the maximum specific growth rate} \\
\varepsilon &= \text{volume fraction occupied by the gas phase}
\end{align*}\]

\[\rho = \text{density (kg/m}^3) \]

\[\mu = \text{maximum specific growth rate (s}^{-1}) \]

\[\mu = \text{coefficient associated with the maximum specific growth rate (s}^{-1}) \]

Subscripts and Superscripts

\[\begin{align*}
\text{air} &= \text{air} \\
f &= \text{gas} \\
\text{matrix} &= \text{matrix} \\
\text{sub} &= \text{substrate} \\
w &= \text{water} \\
x &= \text{microbes} \\
0 &= \text{initial}
\end{align*}\]

Special Symbol

\[\langle \gamma \rangle_{l,1} = \text{intrinsic average} \]

References